Wave Collapse Doesn't Matter

Chris Stucchio, Courant Institute

Joint work with Avy Soffer, Juerg Frohlich and Michael Sigal.

Quantum Mechanics - consensus

Wavefunction

State of the universe

$$\psi(x_1, \ldots, x_N, t)$$

$$\begin{aligned} x_i &= \text{position of } i\text{'th particle} \\ t &= \text{time} \end{aligned}$$

Probability Theory

Probability distribution of particle *configurations*

$$|\psi(x_1, \ldots, x_N, t)|^2 dx_1 \ldots dx_N$$

Evolution

$$i\partial_t \psi = H\psi$$

Schrodinger Equation

Physical Facts

- Suppose we allow the wavefunction evolves to a “split” state

$$\psi(x,T) = \sqrt{\lambda}\phi(x-L) + \sqrt{1-\lambda}\phi(x+L)$$

- Meaning of this wavefunction:

$$\begin{aligned} \text{particle near } x = L & \qquad \text{with probability } \lambda \\ \text{particle near } x = -L & \qquad \text{with probability } 1-\lambda \end{aligned}$$

Repeated “measurements” of particle position will yield same result

Physical Facts

- In the absence of measurement, interference effects are observed.
- Split, recombine than measure:

$$P(x) = |\phi_1(x)|^2 + |\phi_2(x)|^2 + \boxed{2\Re\phi_1(x)\phi_2(x)}$$

interference term

- Split, measure, recombine, then measure again:

$$P(x) = |\phi_1(x)|^2 + |\phi_2(x)|^2$$

Copenhagen Interpretation and Wave Collapse

“Textbook Quantum Mechanics”

How to predict outcome of experiments

- "Prepare" initial wavefunction.

$$\psi(x,0) = \sqrt{\lambda}\phi(x-L) + \sqrt{1-\lambda}\phi(x+L)$$

- Allow it to evolve under Schrodinger equation.

- "Measure" the position of the particle.

How to predict outcome of experiments

$$\sqrt{\lambda}\phi(x-L)+\sqrt{1-\lambda}\phi(x+L)$$

$\text{probability} = \lambda$ $\text{probability} = 1-\lambda$

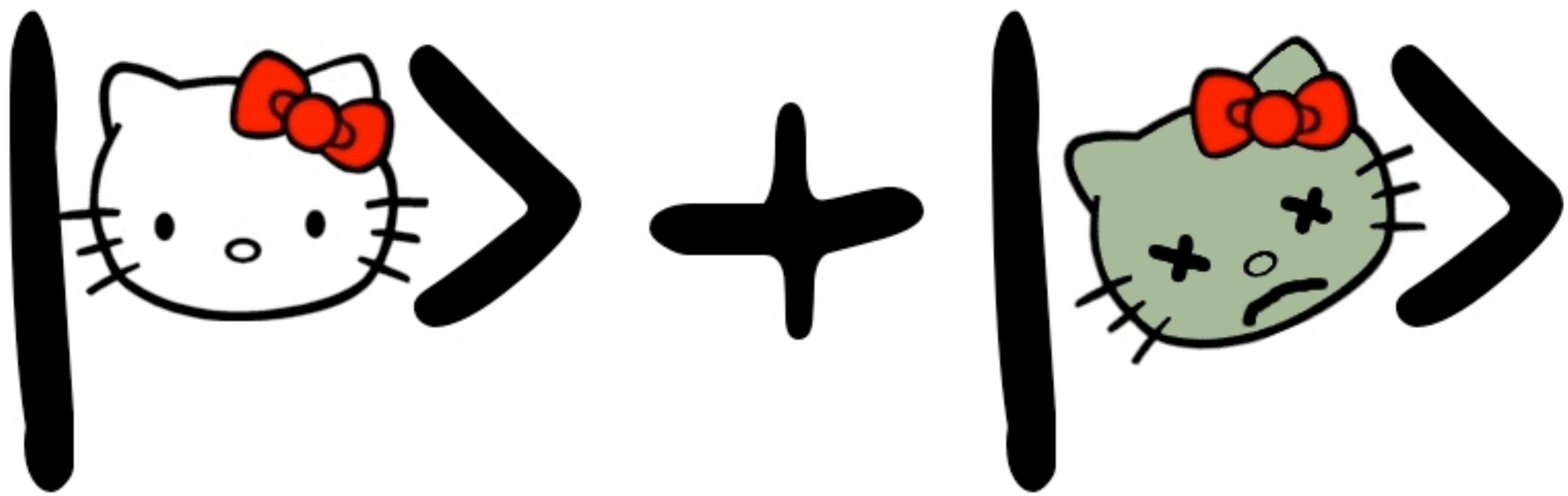
measurement

$\phi(x-L)$ $\phi(x+L)$

Problems with this interpretation

- Why do certain states of the universe constitute a measurement?
- Why are measurements special?
- Are there experiments which are not measurements?

Are all wavefunctions possible?



(Not normalized)

Decoherence

Dynamics in configuration space

Configuration space is really big.

Many particles moving a small distance adds up.

$$|(1,1,\ldots,1)-(0,0,\ldots,0)|=\sqrt{3N}$$

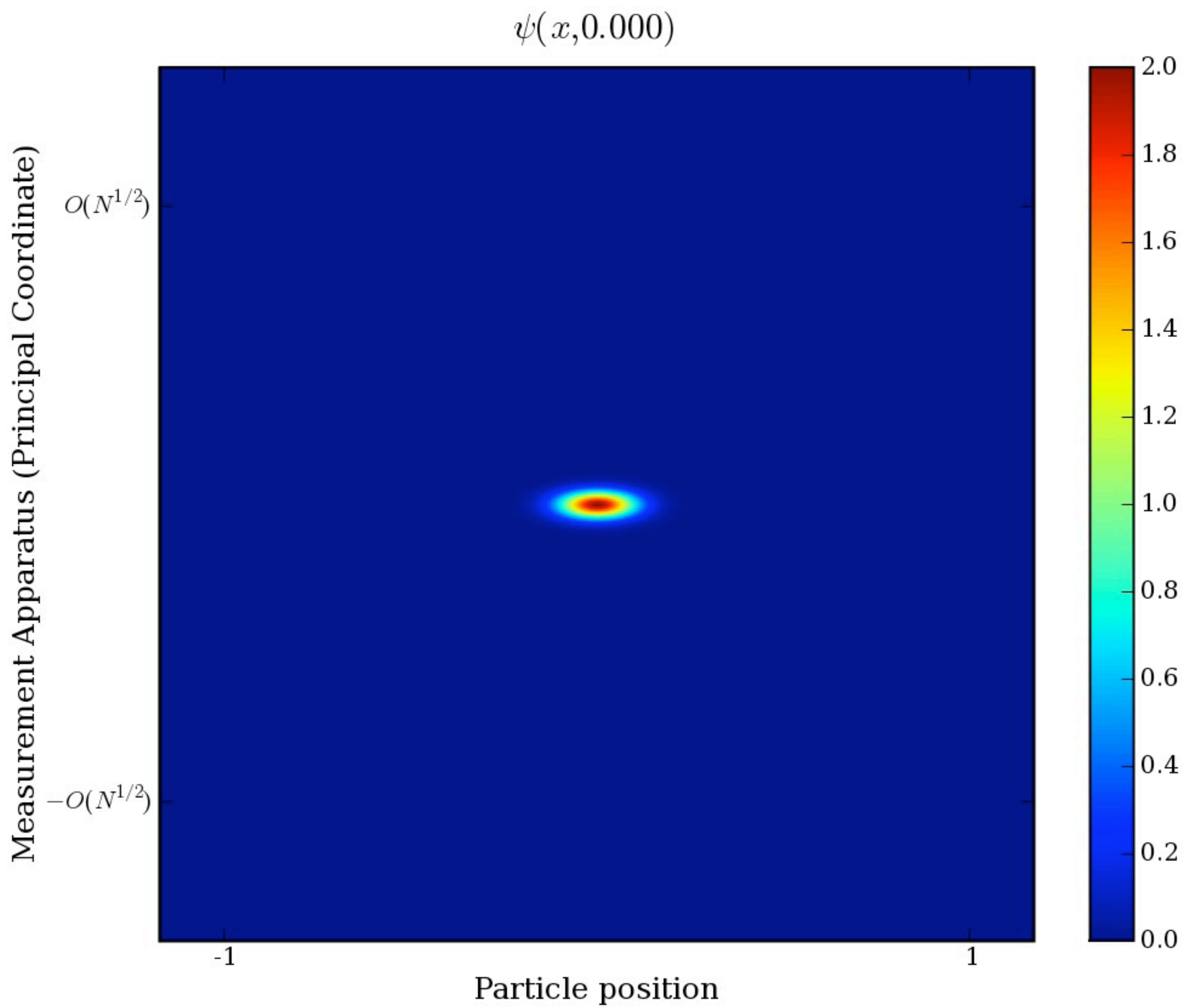
Measurements are not special

- Between measurements, the system remains on a low-dimensional submanifold of configuration space.

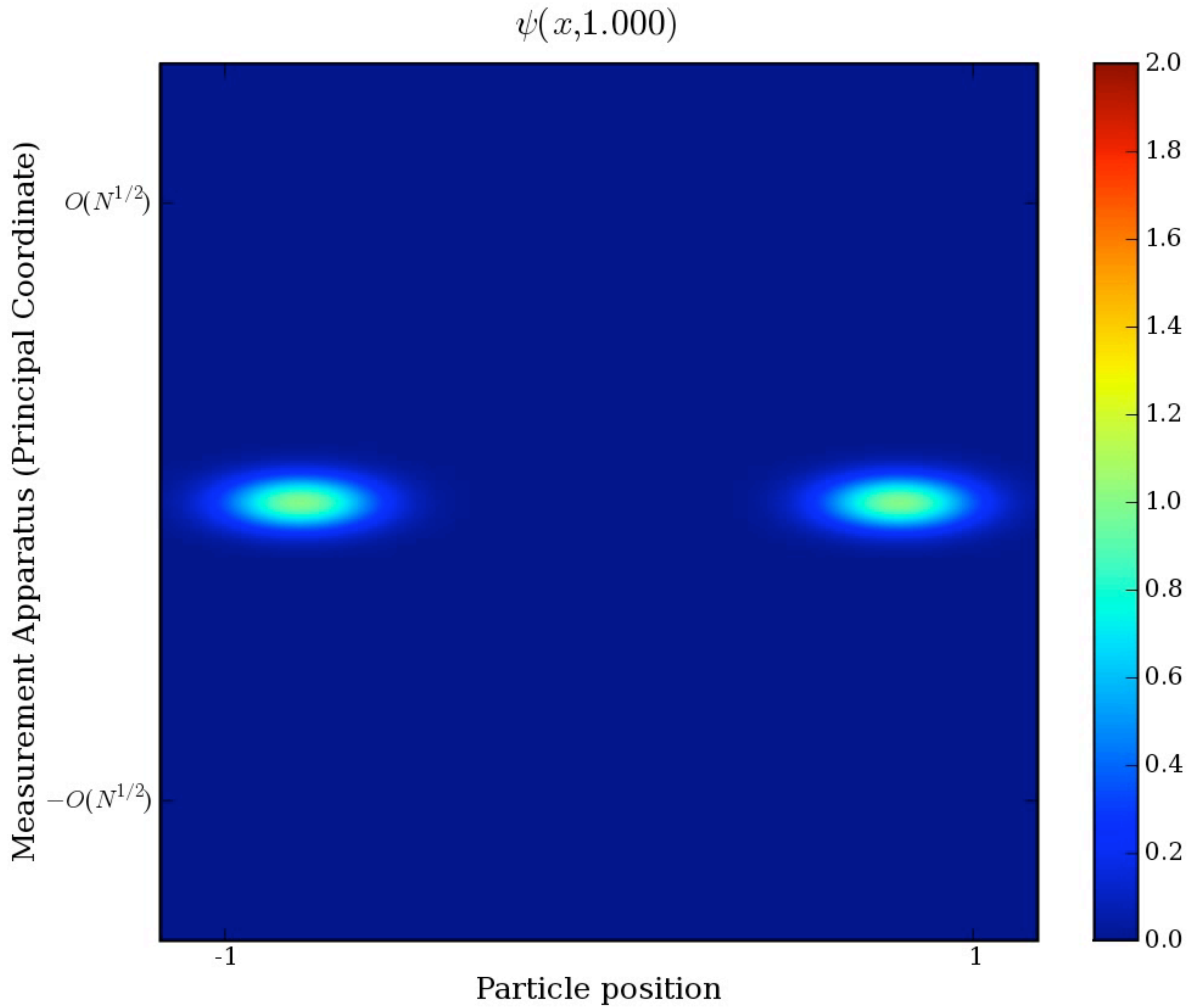
- Measurements are interactions in which many degrees of freedom become relevant.

- After measurement, different states are a distance $O(\sqrt{N})$ apart. This implies no interference, since:

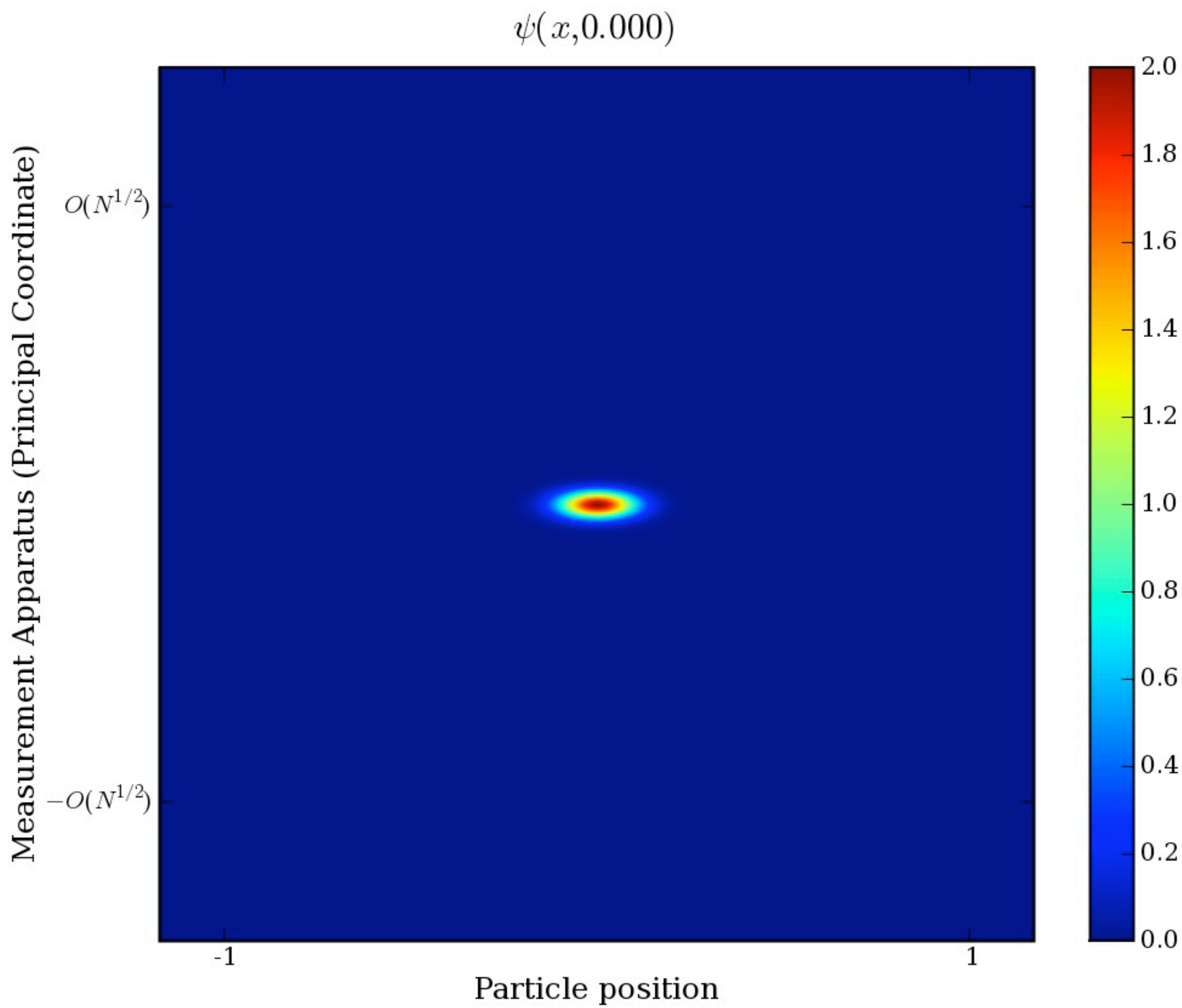
$$2\Re\bar{\psi}_1(x)\psi_2(x) \approx 0$$



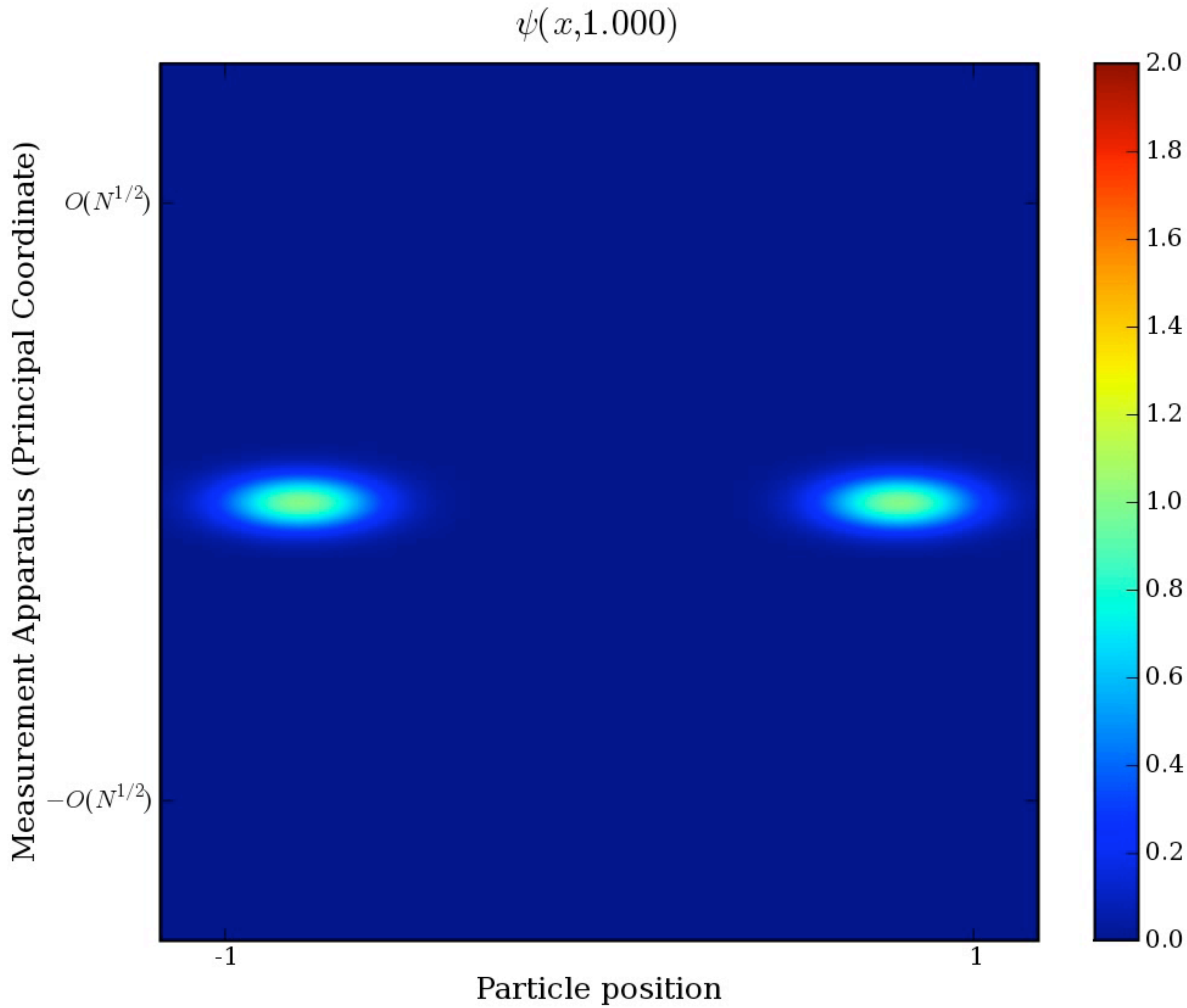
An unmeasured interaction



An unmeasured interaction



The measurement process



The measurement process

Interaction Switched On

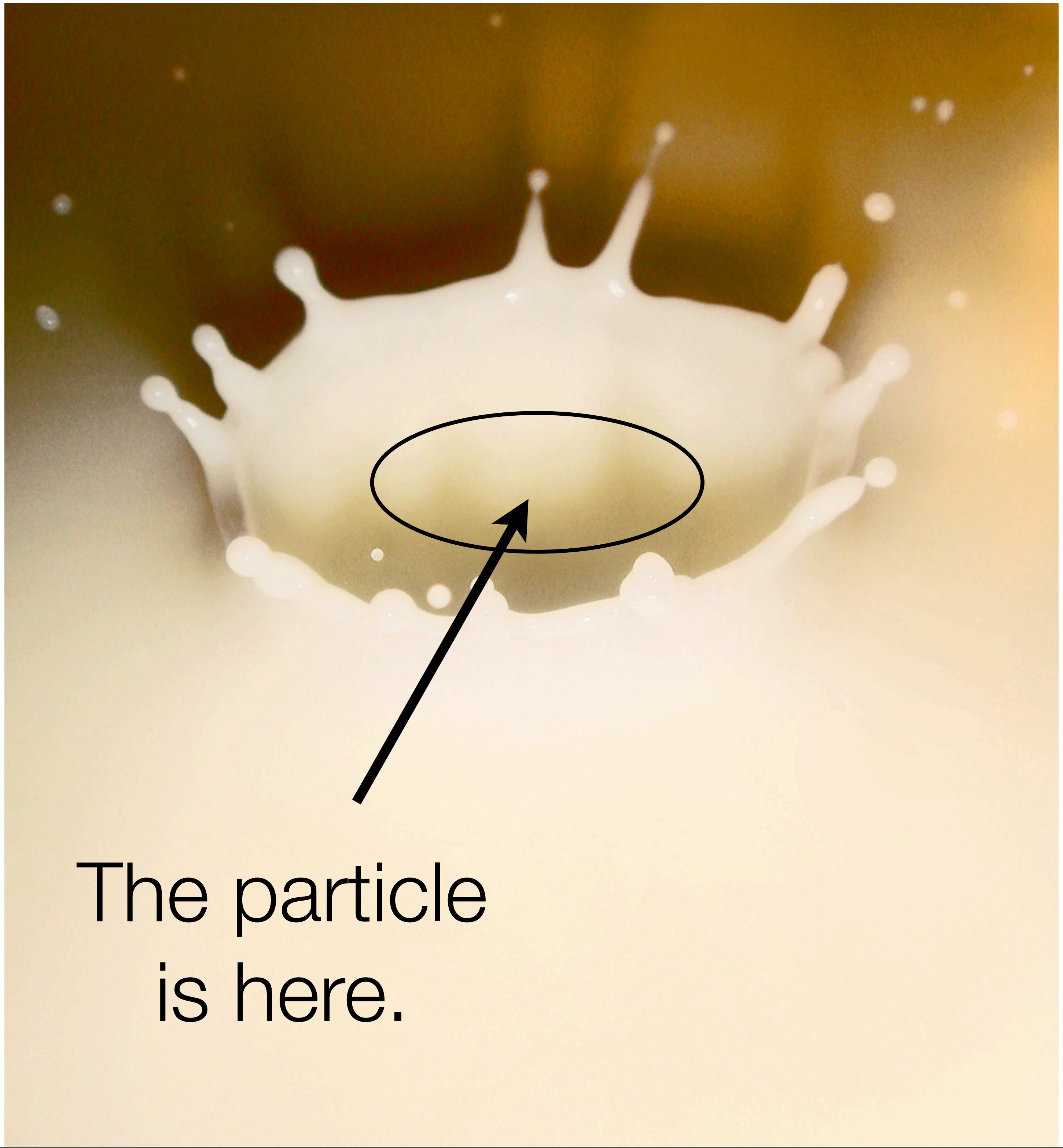
A realistic example

The measurement process

- Want to measure the position of a quantum particle.
- Measurement apparatus is a many-body quantum fluid (BEC), which interacts with particle.
- Use conventional methods to measure position of the splash.

The measurement process

- Want to measure the position of a quantum particle.
- Measurement apparatus is a many-body quantum fluid (BEC), which interacts with particle.
- Use conventional methods to measure position of the splash.



Many Body Schrodinger equation

$$i\partial_t \psi(x,\vec{y},t) = \left[\frac{-\Delta_x}{2M} + \frac{-\Delta_y}{2m} + \sum_j V_p^N(x-y_j) + \frac{1}{2}\sum_{i\neq j} V_s^N(y_i-y_j) \right] \psi(x,\vec{y},t)$$

$$\psi_0(x,y) = \phi(x)\prod_{j=1}^{N}\chi(y_j)$$

$$\begin{aligned} x &= \text{Position of particle to be measured} \\ y_j &= \text{Position of } j\text{-th fluid particle} \\ V_P^N(x-y_j) &= \text{Interaction between particle and fluid} \\ V_S^N(y_i-y_j) &= \text{Internal fluid interaction} \end{aligned}$$

Hydrodynamic Formulation

$$
\begin{aligned}
\partial_t \boldsymbol{\rho}(x,\vec{y},t) + \nabla \cdot [\boldsymbol{\rho}(x,\vec{y},t) v(x,\vec{y},t)] &= 0 \\
\partial_t \vec{\mathbf{v}}(x,\vec{y},t) + \vec{\mathbf{v}}(x,\vec{y},t) \cdot \nabla \vec{\mathbf{v}}(x,\vec{y},t) &= -\widetilde{\nabla} V(x,\vec{y})
\end{aligned}
$$

$$\rho(x,\vec{y},t) = |\psi(x,\vec{y},t)|^2$$

$$\widetilde{\nabla} = (M^{-1}\nabla_x, m^{-1}\nabla_{\vec{y}_1}, \ldots, m^{-1}\nabla_{\vec{y}_N});$$

$$V(x,\vec{y}) = \sum_{j=1}^{N} V_p^N(x - \vec{y}_j) + \frac{1}{2}\sum_{i \neq j} V_s^N(\vec{y}_i - \vec{y}_j)$$

$$+ \frac{\Delta_x \sqrt{\boldsymbol{\rho}(x,\vec{y},t)}}{M\sqrt{\boldsymbol{\rho}(x,\vec{y},t)}} + \frac{\Delta_y \sqrt{\boldsymbol{\rho}(x,\vec{y},t)}}{m\sqrt{\boldsymbol{\rho}(x,\vec{y},t)}}$$

Multiconfiguration Reduction

- Many-Body Schrodinger equation is hard. Solution: derive reduced equation.
- Reduced variables $\rho(x, y_1, t), v_x(x, y_1, t)$ and $v_y(x, y_1, t)$

$$\boldsymbol{\rho}(x, \vec{y}, t) \quad = \quad \prod_{j=1}^{N} \rho(x, \vec{y}_j, t)$$

$$\vec{\mathbf{v}}(x, , t) = \left[\sum_{j=1}^{N} v_x(x, y_j, t), v_y(x, y_1, t), \ldots, v_y(x, y_N, t) \right]$$

Derivation of reduced equation

$$\begin{aligned}\Bigg(\partial_t \rho(x,\vec{y}_j,t) + \left[\rho(x,\vec{y}_j,t)\sum_{l=1}^{N}\vec{v}_x(x,\vec{y}_l,t)\right]\nabla_x \cdot \rho(x,\vec{y}_j,t) \\ + \frac{1}{N}\rho(x,\vec{y}_j,t)\nabla_x \cdot \left[\rho(x,\vec{y}_j,t)\sum_{l=1}^{N}\vec{v}_x(x,\vec{y}_l,t)\right] \\ + \nabla_{y_j} \cdot [\rho(x,\vec{y}_j,t)\vec{v}_y(x,\vec{y}_j,t)]\Bigg) = 0\end{aligned}$$

Derivation of reduced equation

$$\Bigg(\partial_t \rho(x,\vec{y}_j,t) + \left[\rho(x,\vec{y}_j,t)\boxed{\sum_{l=1}^{N}\vec{v}_x(x,\vec{y}_l,t)}\right]\nabla_x \cdot \rho(x,\vec{y}_j,t)$$

$$+\frac{1}{N}\rho(x,\vec{y}_j,t)\nabla_x \cdot \left[\rho(x,\vec{y}_j,t)\boxed{\sum_{l=1}^{N}\vec{v}_x(x,\vec{y}_l,t)}\right]$$

$$+\nabla_{y_j}\cdot\left[\rho(x,\vec{y}_j,t)\vec{v}_y(x,\vec{y}_j,t)\right]\Bigg) = 0$$

We will reduce this

Derivation of reduced equation

- Equation for velocities:

$$\sum_{j=1}^{N}\left[\partial_t \vec{v}_x(x,\vec{y}_j,t) + \left(\sum_{k=1}^{N}\vec{v}_x(x,\vec{y}_k,t)\right)\cdot\nabla_x \vec{v}_x(x,\vec{y}_j,t) \right.$$
$$\left. + \vec{v}_y(x,\vec{y}_j,t)\cdot\nabla_{y_j}\vec{v}_x(x,\vec{y}_j,t)\right] = -\frac{\nabla_x}{M}V(x,\vec{y})$$

$$\partial_t \vec{v}_y(x,\vec{y}_j,t) + \left(\sum_{k=1}^{N}\vec{v}_x(x,\vec{y}_k,t)\right)\cdot\nabla_x \vec{v}_y(x,\vec{y}_j,t)$$
$$+ \vec{v}_y(x,\vec{y}_j,t)\cdot\nabla_y \vec{v}_y(x,\vec{y}_j,t) = -\frac{\nabla_{\vec{y}_j}}{m}V(x,\vec{y})$$

Derivation of reduced equation

- Equation for velocities:

$$\sum_{j=1}^{N}\left[\partial_t \vec{v}_x(x,\vec{y}_j,t) + \left(\boxed{\sum_{k=1}^{N}\vec{v}_x(x,\vec{y}_k,t)}\right)\cdot\nabla_x \vec{v}_x(x,\vec{y}_j,t) + \vec{v}_y(x,\vec{y}_j,t)\cdot\nabla_{y_j}\vec{v}_x(x,\vec{y}_j,t)\right] = -\frac{\nabla_x}{M}V(x,\vec{y})$$

$$\partial_t \vec{v}_y(x,\vec{y}_j,t) + \left(\boxed{\sum_{k=1}^{N}\vec{v}_x(x,\vec{y}_k,t)}\right)\cdot\nabla_x \vec{v}_y(x,\vec{y}_j,t) + \vec{v}_y(x,\vec{y}_j,t)\cdot\nabla_y \vec{v}_y(x,\vec{y}_j,t) = -\frac{\nabla_{\vec{y}_j}}{m}V(x,\vec{y})$$

Mean field for velocity

- We need not consider a general point in configuration space, only typical ones.

- Probability distribution of fluid particle:

$$\frac{\rho(x, y_j, t)}{\int \rho(x, y_j, t) dy_j} dy_j$$

- Central limit theorem:

$$\sum_{j=2}^{N} v_x(x, y_j, t) \to (N-1) \frac{\int v_x(x, y, t) \rho(x, y, t) dy}{\int \rho(x, y, t) dy}$$

Mean Field for Potential

$$V(x,\vec{y}) = \sum_{j=1}^{N} V_p^N(x-\vec{y}_j) + \frac{1}{2}\sum_{i\neq j} V_s^N(\vec{y}_i-\vec{y}_j)$$
$$+ \frac{\Delta_x\sqrt{\boldsymbol{\rho}(x,\vec{y},t)}}{M\sqrt{\boldsymbol{\rho}(x,\vec{y},t)}} + \frac{\Delta_y\sqrt{\boldsymbol{\rho}(x,\vec{y},t)}}{m\sqrt{\boldsymbol{\rho}(x,\vec{y},t)}}$$

- Similar tricks can be used for the potential:

$$\sum_{j\neq 1} V_s^N(y_1-y_j) \to (N-1)\frac{\int V_s^N(y_1-y)\rho(x,y,t)dy}{\int \rho(x,y,t)dy}$$

Probably Approximately Correct

- How accurate is this?

- Mcdiarmid's inequality. For a vector of i.i.d. variables, if

$$\sup_{x,\hat{x}_i} |f(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, x_N) - f(x_1, \ldots, x_{i-1}, x_i, \hat{x}_{i+1}, x_N)| < C$$

- then:

$$P(|f(\vec{x}) - E[f(\vec{x})]| > \epsilon) \leq 2 \exp\left(-\frac{2\epsilon^2}{NC^2}\right)$$

Probably Approximately Correct

- Implication:

$$P\left(|\sum_{j=1}^{N} V_p^N(x-y_j)-(N-1)\frac{\int V_p^N(x-y)\rho(x,y)dy}{\int \rho(x,y)dy}| \geq N\epsilon\right)$$
$$\leq 2\exp\left(-\frac{2\epsilon^2 N}{||V_p^N(x)||_{L^\infty}}\right)$$

- The probability distribution is w.r.t. conditional distribution of fluid particle:

$$P(y)=\frac{\rho(x,y)}{\int \rho(x,y)dy}$$

Y-indepence of equations

- Equations depend (to order O(N)) not on $v_x(x, y, t)$ but on it's expected value:

$$
\begin{aligned}
\partial_t \rho(x,y,t) + (N-1)(\nabla_x \cdot \rho(x,y,t)) & \boxed{\left[\frac{\int \vec{v}_x(x,y',t)\rho(x,y',t)dy'}{\int \rho(x,y',t)dy'} d\vec{y}_l \right]} \\
+ (N-1)\rho(x,y,t)\nabla_x \cdot & \boxed{\left[\frac{\int \vec{v}_x(x,y',t)\rho(x,y',t)dy'}{\int \rho(x,y',t)dy'} d\vec{y}_l \right]} \\
+ \nabla_x \cdot [\rho(x,y,t)\vec{v}(x,y,t)] + \nabla_y [\rho(x,y,t)\vec{v}_y(x,y,t)] & = 0
\end{aligned}
$$

Y-indepence of equations

- Equations depend (to order O(N)) not on $v_x(x,y,t)$ but on it's expected value:

$$\sum_{j=1}^{N}\left[\partial_t \vec{v}_x(x,\vec{y}_j,t) + (N-1)\boxed{\left[\frac{\int \vec{v}_x(x,y',t)\rho(x,y',t)dy'}{\int \rho(x,y',t)dy'}\right]} \cdot \nabla_x \vec{v}_x(x,\vec{y}_j,t)\right.$$

$$\left. + \vec{v}_x(x,\vec{y}_j,t)\cdot\nabla_x\vec{v}_x(x,\vec{y}_j,t) + \vec{v}_y(x,\vec{y}_j,t)\cdot\nabla_y\vec{v}_x(x,\vec{y}_j,t)\right]$$

$$= \sum_{j=1}^{N}\left(-\frac{\nabla_x}{M}V_p^N(x-\vec{y}_j)\right) - \frac{\nabla_x}{M}V_q$$

Partial Y-indepence of equations

- Equations depend (to order O(N)) not on $v_x(x, y, t)$ but on it's expected value.

- We can therefore consider expected value of x-velocity a reduced variable:

$$\tilde{\vec{v}}_x(x,t) = (N-1)\frac{\int \vec{v}_x(x,y',t)\rho(x,y',t)dy'}{\int \rho(x,y',t)dy'}$$

- Derive equation for this reduced variable by multiplying velocity equation by probability distribution, and integrate by parts.

Plug and chug

$$\begin{aligned}
\partial_t \vec{v}_x(x,y,t) + \tilde{\vec{v}}_x(x,t) \cdot \nabla_x \vec{v}_x(x,y,t) + \vec{v}_y(x,y,t) \cdot \nabla_y \vec{v}_x(x,y,t) \\
= -\frac{\nabla_x}{M} V_p^N(x-y) - \frac{\nabla_x}{M^2} \frac{\Delta_x \rho^{1/2}(x,y,t)}{\rho^{1/2}(x,y,t)} - \frac{\nabla_x}{Mm} \frac{\Delta_y \rho^{1/2}(x,y,t)}{\rho^{1/2}(x,y,t)} \\
- \frac{2(N-1)\nabla_x}{M^2} \frac{\nabla_x \rho^{1/2}(x,y,t)}{\rho^{1/2}(x,y,t)} \cdot \frac{\int \rho^{1/2}(x,y',t) \nabla_x \rho^{1/2}(x,y',t) dy'}{\int \rho(x,y'',t) dy''}
\end{aligned}$$

Plug and chug

$$\int [\partial_t \vec{v}_x(x,y,t) + \tilde{\vec{v}}_x(x,t) \cdot \nabla_x \vec{v}_x(x,y,t) + \vec{v}_y(x,y,t) \cdot \nabla_y \vec{v}_x(x,y,t)] f(x,y,t) dy$$

$$= \partial_t \tilde{\vec{v}}_x(x,t) + \tilde{\vec{v}}_x(x,t) \cdot \nabla_x \tilde{\vec{v}}_x(x,t)$$

$$+ \int \vec{v}_x(x,y,t)$$

$$\times \Big(\partial_t f(x,y,t) + [\tilde{\vec{v}}_x(x,t) \cdot \nabla_x f(x,y,t)] + [\nabla_y \cdot \vec{v}_y(x,y,t) + \vec{v}_y(x,y,t) \cdot \nabla_y f(x,y,t)] \Big) dy$$

$$= \partial_t \tilde{\vec{v}}_x(x,t) + \tilde{\vec{v}}_x(x,t) \cdot \nabla_x \tilde{\vec{v}}_x(x,t)$$

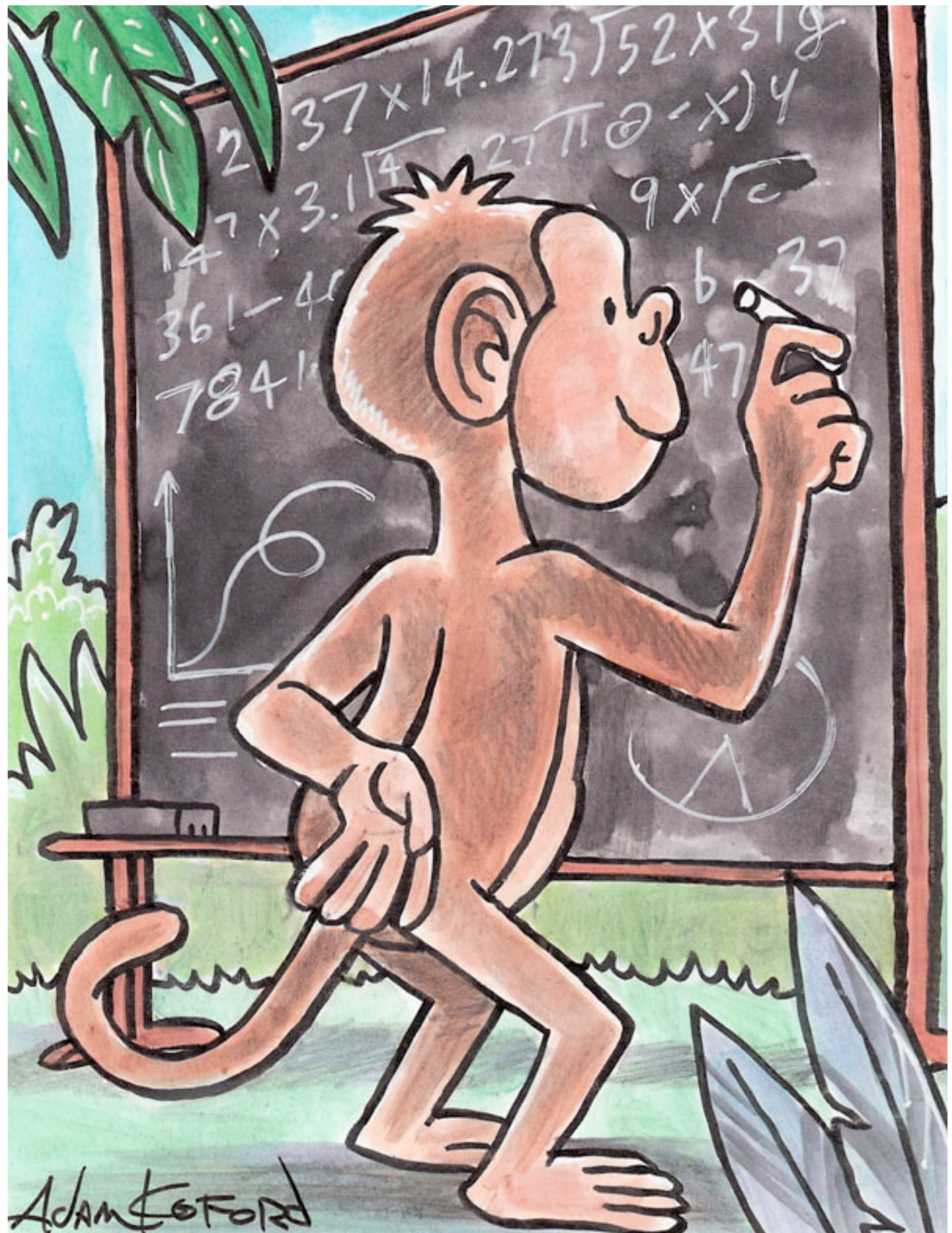
Plug and chug

$$\sum_{j=1}^{N}\sum_{k\neq j}^{N}\nabla_x\rho^{1/2}(x,\vec{y}_j,t)\cdot\nabla_x\rho^{1/2}(x,\vec{y}_k,t)\prod_{l\neq j,k}\rho^{1/2}(x,\vec{y}_l,t)$$

$$=\sum_{j=1}^{N}2\nabla_x\rho^{1/2}(x,\vec{y}_j,t)\cdot\left(\sum_{k\neq j}^{N}\frac{\nabla_x\rho^{1/2}(x,\vec{y}_k,t)}{\rho^{1/2}(x,\vec{y}_1,t)}\prod_{l\neq j}\rho^{1/2}(x,\vec{y}_l,t)\right)$$

$$=\sum_{j=1}^{N}2\nabla_x\rho^{1/2}(x,\vec{y}_j,t)\cdot\left(\sum_{k\neq j}^{N}\frac{\nabla_x\rho^{1/2}(x,\vec{y}_k,t)}{\rho^{1/2}(x,\vec{y}_1,t)}\right)\prod_{l\neq j}\rho^{1/2}(x,\vec{y}_l,t)$$

Plug and chug



Reduced Equations

- One more substitution: $\rho(x, y, t) = P^{1/N}(x, t) f(x, y, t).$
- $P(x, t)$ -- Probability distribution of particle position
- $f(x, y, t)$ -- Fluid distribution assuming particle is at x.
- Note: $f(x, y, t) = \dfrac{\rho(x, y, t)}{\int \rho(x, y, t) dy}$

Reduced Equations

$$
\begin{aligned}
\partial_t P(x,t) + \nabla_x \cdot [\tilde{\vec{v}}_x(x,t) P(x,t)] &= 0 \\
\partial_t f(x,y,t) + \tilde{\vec{v}}_x(x,t) \cdot \nabla_x f(x,y,t) + \nabla_y \cdot [\vec{v}_y(x,y,t) f(x,y,t)] &= 0
\end{aligned}
$$

$$
\left(\partial_t \tilde{\vec{v}}_x(x,t) + \tilde{\vec{v}}_x(x,t) \cdot \nabla_x \tilde{\vec{v}}_x(x,t)\right) = -\frac{(N-1)\nabla_x}{M} \int \left[V(x-y) + V_q(x,y,t)\right] f(x,y,t) dy
$$

$$
\begin{aligned}
&\partial_t \vec{v}_y(x,y,t) + \tilde{\vec{v}}_x(x,t) \cdot \nabla_x \vec{v}_y(x,y,t) + \vec{v}_y(x,y,t) \cdot \nabla_y \vec{v}_y(x,y,t) \\
&= -\frac{\nabla_y}{m} V_p^N(x-y) + \frac{\nabla_y}{m}(N-1) \int V_s^N(y-y') f(x,y',t) dy' - \frac{\nabla_y}{m} V_q(x,y,t)
\end{aligned}
$$

Scaling

- Work on finite box with fixed particle density, let box get bigger.

$$N/|\Lambda| = \rho_0, \Lambda \uparrow \mathbb{R}^3$$

- Scale particle and two-body fluid force with N:

$$M \sim MN, \quad V_s^N(y-y') = N^{-1}V_s(y-y')$$

- With this scaling, a long calculation shows that X-components of quantum pressure also vanish.

- Scaling reasonable: physical examples have M=235 or M=720, m=4.

Scaling

$$\begin{aligned}\partial_t P(x,t) + \nabla_x \cdot [\tilde{\vec{v}}_x(x,t)P(x,t)] &= 0 \\ \partial_t f(x,y,t) + \tilde{\vec{v}}_x(x,t)\cdot\nabla_x f(x,y,t) + \nabla_y \cdot [\vec{v}_y(x,y,t)f(x,y,t)] &= 0\end{aligned}$$

$$(\partial_t \tilde{\vec{v}}_x(x,t) + \tilde{\vec{v}}_x(x,t)\cdot\nabla_x\tilde{\vec{v}}_x(x,t)) = -\frac{\nabla_x}{M}\int V(x-y)f(x,y,t)dy$$

$$\partial_t\vec{v}_y(x,y,t) + \tilde{\vec{v}}(x,t)\cdot\nabla_x\vec{v}_y(x,y,t) + \vec{v}_y(x,y,t)\cdot\nabla_y\vec{v}_y(x,y,t)$$

$$= -\frac{\nabla_y}{m}V(x-y) + \frac{\nabla_y}{m}\int V_s(y-y')f(x,y',t)dy' - \frac{\nabla_y}{m}V_q(x,y,t)$$

Bohmian Coordinates

- Equation of characteristics:

$$\begin{aligned} q'(x,t) &= \tilde{\vec{v}}_x(x,t) \\ q(x,0) &= x \end{aligned}$$

- Result along characteristic:

$$\partial_t f(q(x,t),y,t) + \nabla_y \cdot [f(q,y,t)\vec{v}_y(q,y,t)] = 0$$

$$\partial_t \vec{v}_y(q,y,t) + \vec{v}_y(q,y,t) \cdot \nabla_y \vec{v}_y(q,y,t)$$
$$= -\frac{\nabla_y}{m} V(q(x,t)-y) + \int V_s(y-y')f(x,y',t)dy - \frac{\nabla_y}{m} V_q(q,y,t)$$

$$q''(x,t) = \frac{-\nabla_y}{M} \int V(q(x,t)-y) f(q(x,t),y,t)dy$$

Equivalent Schrodinger Equation

- Equivalent to NLS coupled to a classical particle.

$$i\partial_t \Psi(y,t) = \left[\frac{-1}{2m}\Delta_y + V(y - q(x,t)) + \int V_s(y-y')|\Psi(y',t)|^2 dy'\right]\Psi(y,t)$$

$$q''(x,t) = -\frac{\nabla_y}{M}\int V(y - q(x,t))|\Psi(y,t)|^2 dy$$

Dynamics: Friction and stopping

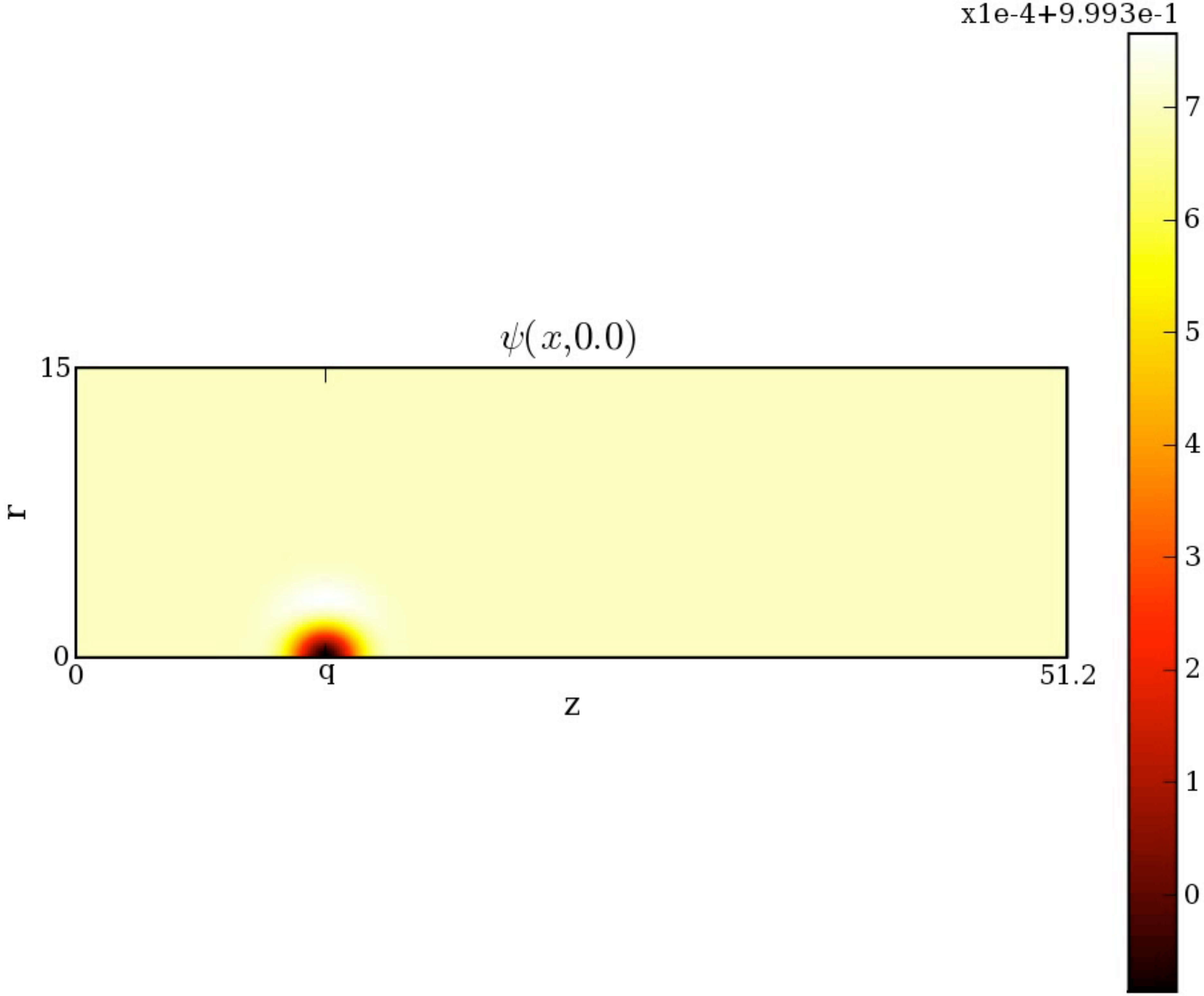
Friction by Cerenkov Radiation

- Particle moves in fluid, and generates a wake behind it. Loss of energy to wake slows the particle down, and is a frictional force.

- If the nonlinear forces are zero, we can prove rigorously that the particle stops in the absence of nonlinear fluid forces. Numerical results confirm result is true for nonlinear fluids.

- Decay rate:

$$
\begin{aligned}
|q'(x,t)| &\leq C\langle t\rangle^{-3/2}) \\
||\nabla_y f(x,y,t) - \nabla_y f(x,y,t=\infty)||_{L^3} &\leq C\langle t\rangle^{-1/2}
\end{aligned}
$$

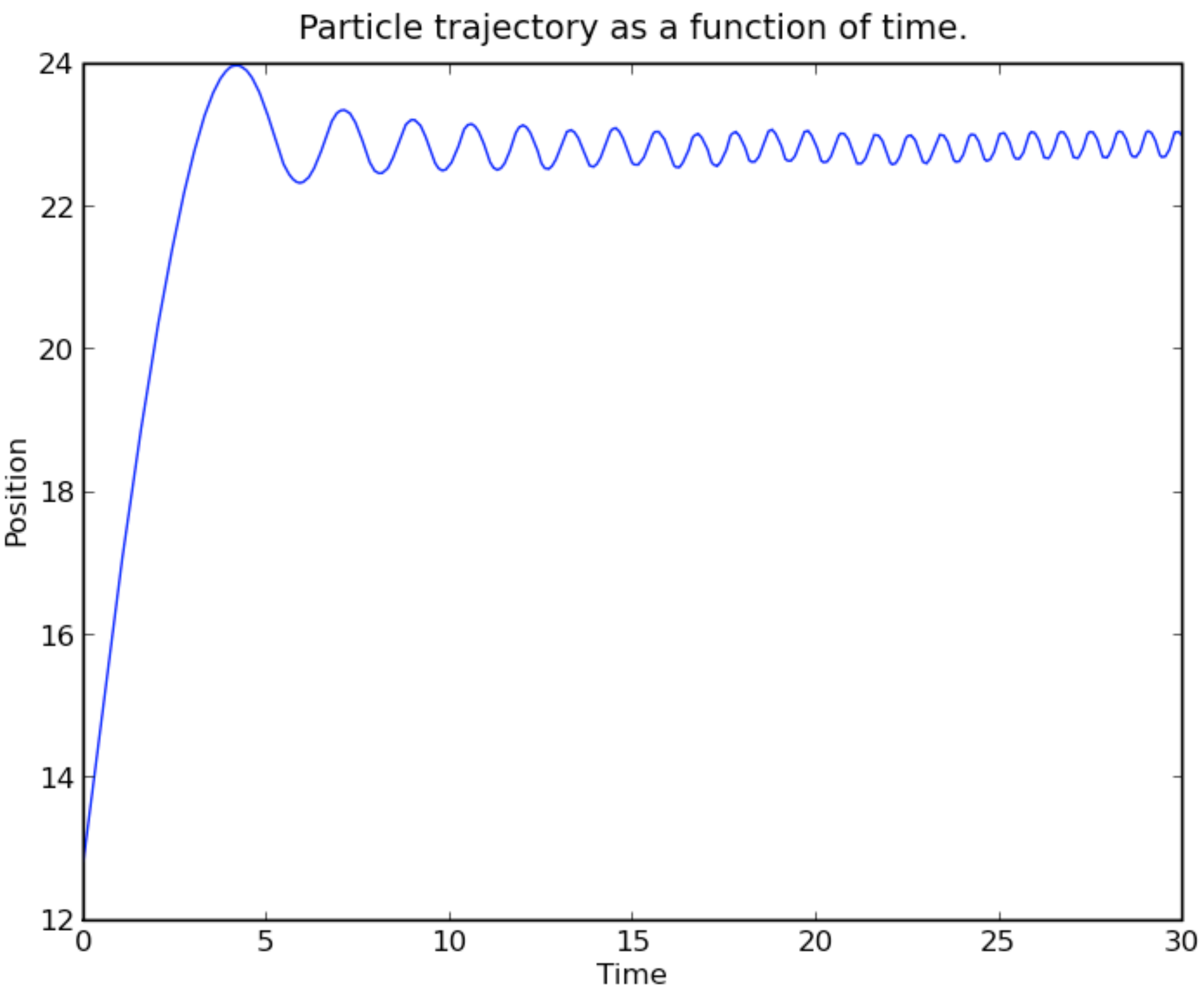


Numerical Results

Repulsive Potential

Numerical results

- Particle eventually stops, but oscillates around it's stopping point.
- Oscillation frequency and decay rate can be calculated (to leading order) by Laplace transforms.

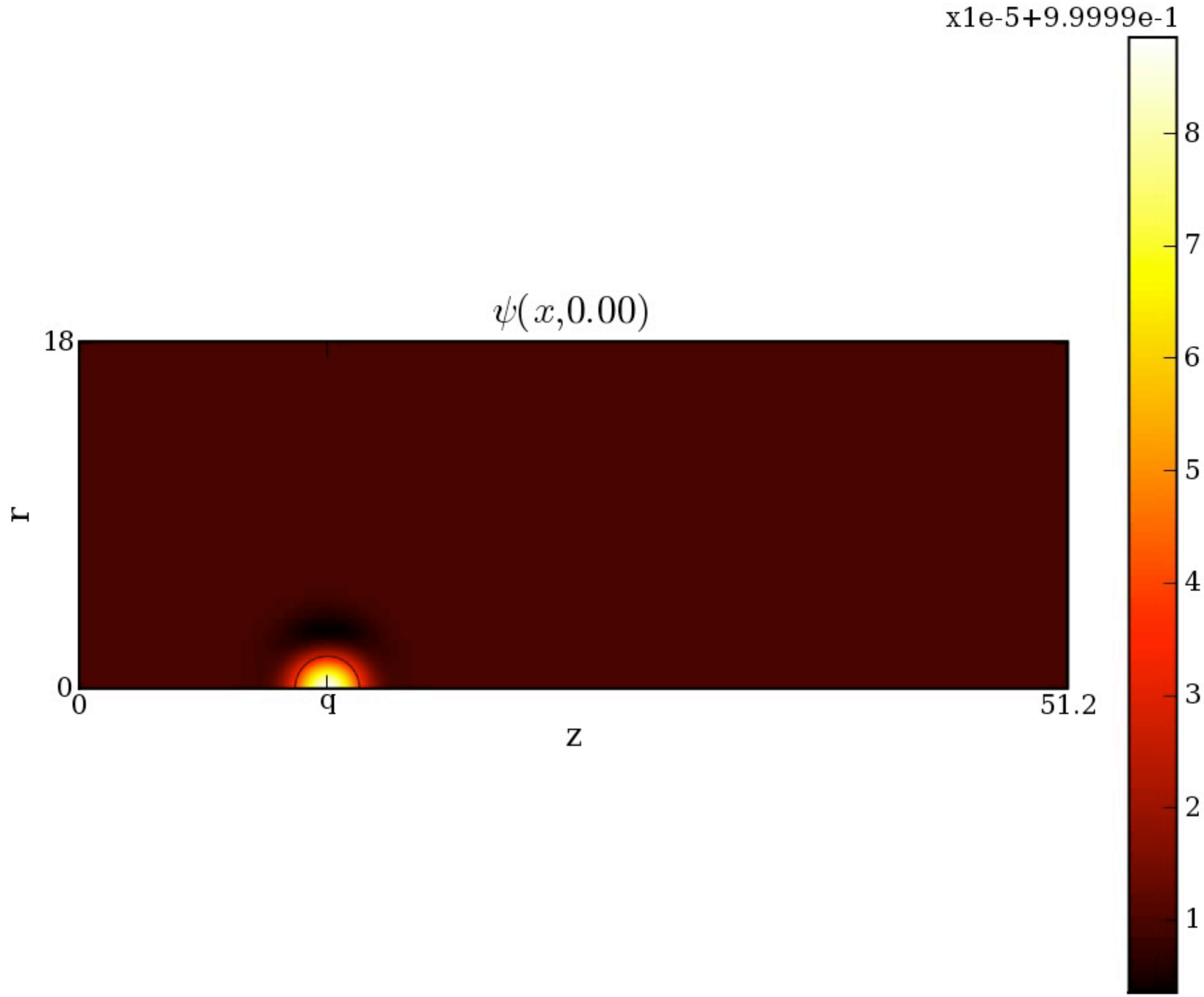


Attractive interactions

- The mass held by an attractive potential will grow without bound, unless arrested by a repulsive nonlinearity.

- Regardless of M, the particle combined with the cloud of particles it attracts will be $O(N)$.

- Semiclassical dynamics are achieved regardless of the mass of the particle!



Numerical Results

Attractive Potential

Key ideas of proof

- Write equation for $q'(x,t)$ to leading order as an *linear* integral equation (which is history dependent):

$$
\begin{aligned}
q''(x,t) &= -\int_0^t K(t,s) q'(x,s) ds + \text{remainder} \\
K(t,s) &= \frac{2\rho_0}{M} \Re \left\langle \partial_{y_z} V(y) \middle| \frac{e^{i\Delta(t-s)/2m} \partial_{y_z}}{-\Delta/2m + V(y)} V(y) \right\rangle
\end{aligned}
$$

- Use dispersive estimates to show that $q'(x,t)$ vanishes, and show remainder does not cause problems.

- Transients appear to leading order in this framework. They can be calculated by dropping the remainder, taking the Laplace transform and searching for poles.

Decoherence

Bringing it back to the wavefunction

- Fix an initial state for the particle, with L larger than the stopping distance.

$$\phi_0(x) = \sqrt{\lambda}\phi(x - L) + \sqrt{1-\lambda}\phi(x + L)$$

- Initial wavefunction:

$$\psi_0(x, \vec{y}) = \phi_0(x) \prod_{j=1}^{N} \chi_0(y_j)$$

Bringing it back to the wavefunction

- Final wavefunction:

$$\psi(x, \vec{y}, t \approx \infty)$$
$$= \sqrt{\lambda}\tilde{\phi}(x - L) \prod_{j=1}^{N} \chi_\infty(y_j - L) + \sqrt{1-\lambda}\tilde{\phi}(x + L) \prod_{j=1}^{N} \chi_\infty(y_j + L)$$

- A “schrodingers cat” wavefunction.

A model for measurement

- Measurement consists of determining the state of the macroscopic system, in this case a vector $\vec{y}$ (or at least some function $F(\vec{y})$).

- From $\vec{y}$ we infer a value for x. But with what statistical significance can we do this?

- This framework covers the instrumentalist picture, Bohmian Mechanics, and most particle-based ontologies.

Statistical Significance

- Consider measurement process: given knowledge of $\vec{y}$, determine value of x. With what statistical significance can we answer this question?

- Partition configuration space $\mathbb{R}^{3N} = \Omega_1 \cup \Omega_2$, and use the rule $x \approx -L$ for $\vec{y} \in \Omega_1$ and vice versa.

- Confidence level: $P_1(\vec{y} \in \Omega_1) + P_2(\vec{y} \in \Omega_2)$, where

$$
\begin{aligned}
dP_1 &= \prod_{j=1}^{N} |\chi_\infty(y_j + L)|^2 d\vec{y} \\
dP_2 &= \prod_{j=1}^{N} |\chi_\infty(y_j - L)|^2 d\vec{y}
\end{aligned}
$$

Statistical Significance

- Choose Ω_1, Ω_2 so that $P_1(\vec{y} \in \Omega_1) = P_2(\vec{y} \in \Omega_2) = p/2$ to get an unbiased estimator.
- This gives best possible decision procedure.
- In the event we know only $F(\vec{y})$ rather than $\vec{y}$, our statistical confidence can only go down.
- $F(\vec{y})$ models deterministic experimental errors, e.g. differences in $\vec{y}$ which are experimentally invisible.

Bounds on the interference:

- Interference term: $2\Re \prod_{j=1}^{N} \bar{\chi}_\infty(y_j + L) d\vec{y} \prod_{j=1}^{N} \chi_\infty(y_j - L)$
- Bounds:

$$\int \left| \prod_{j=1}^{N} \chi_\infty(y_j + L) \prod_{j=1}^{N} \chi_\infty(y_j - L) \right| d\vec{y}$$

$$\leq || \prod_{j=1}^{N} \chi_\infty(y_j + L) ||_{\Omega_1} || \prod_{j=1}^{N} \chi_\infty(y_j - L) ||_{\Omega_1}$$

$$+ || \prod_{j=1}^{N} \chi_\infty(y_j + L) ||_{\Omega_2} || \prod_{j=1}^{N} \chi_\infty(y_j - L) ||_{\Omega_2}$$

$$\leq \sqrt{p/2} 1 + 1 \sqrt{p/2} = O(\sqrt{\text{statistical confidence}})$$

Statistical Significance and Interference

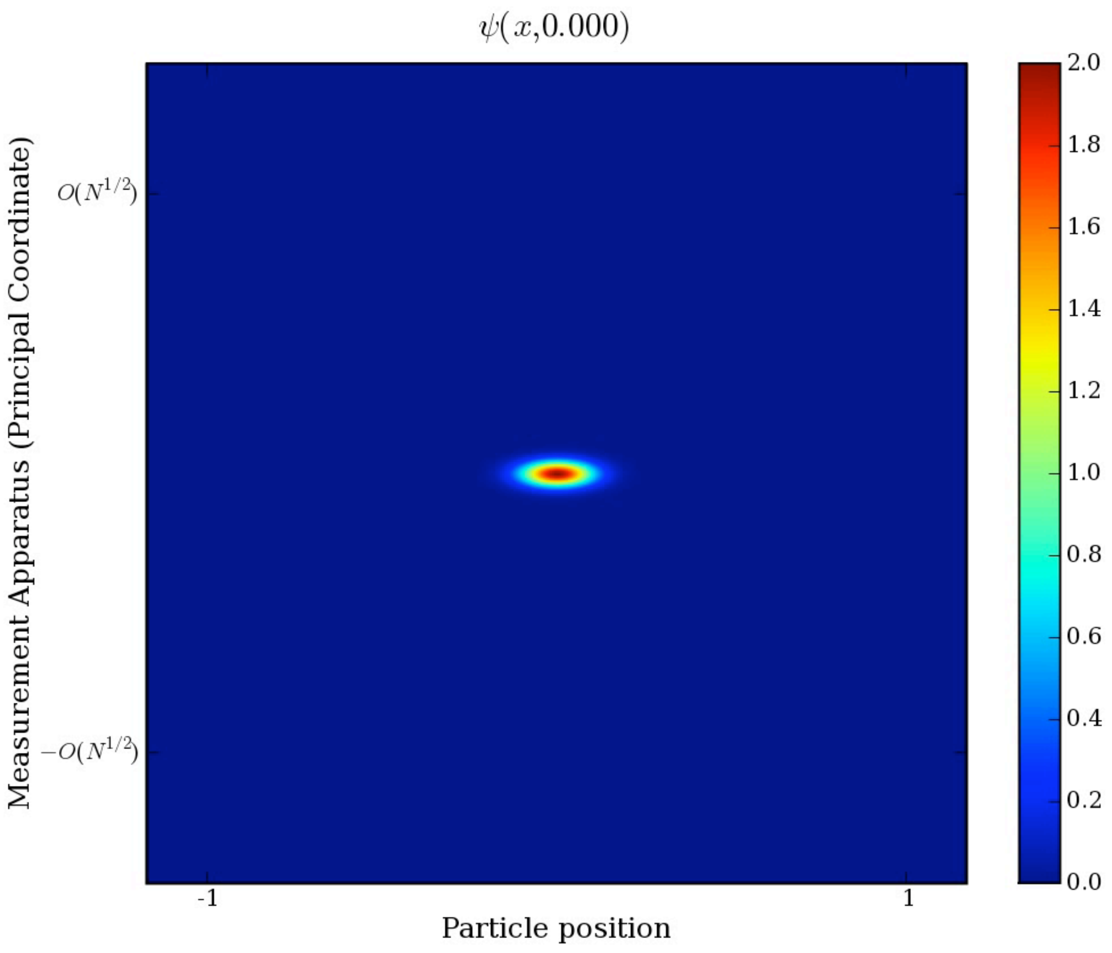
- The p-value of the experiment provides an upper bound on the size of the interference term.
- Good experiments (statistically significant ones) destroy interference.
- Experimental prediction: “fractional measurements” are possible. A “fractional measurement” is an experiment with large p-values which only partially destroys interference.

The One-Pixel Camera

- Consider an experimental measurement consisting of counting the number of fluid particles in a fixed region (the “pixel”).
- If splash is contained within pixel, average number of fluid particles observed is different than if not. This provides a means of determining whether the particle is within the pixel.
- Statistical significance: p=0.1% requires splash to involve 47 particles for repulsive particle previously simulated.
- Thus, 47 fluid particles is sufficient to reduce interference to about 5% of the total wavefunction.

The Wave Collapse Approximation

- Suppose we make a measurement, and the particle is observed to be on the right.
- To simplify calculations, set left wavepacket equal to zero.
- This is computationally simpler than tracking both wavepackets, and equally accurate.



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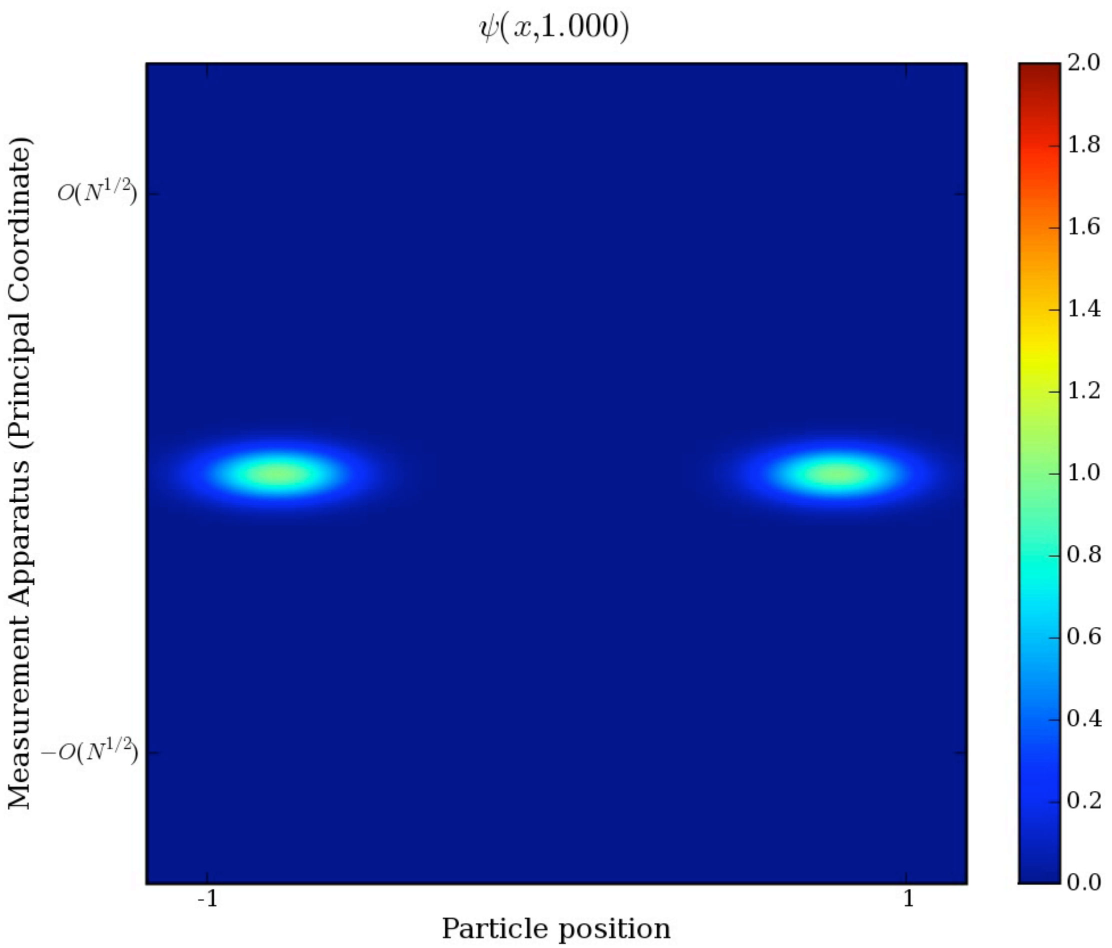


Interpreting the results

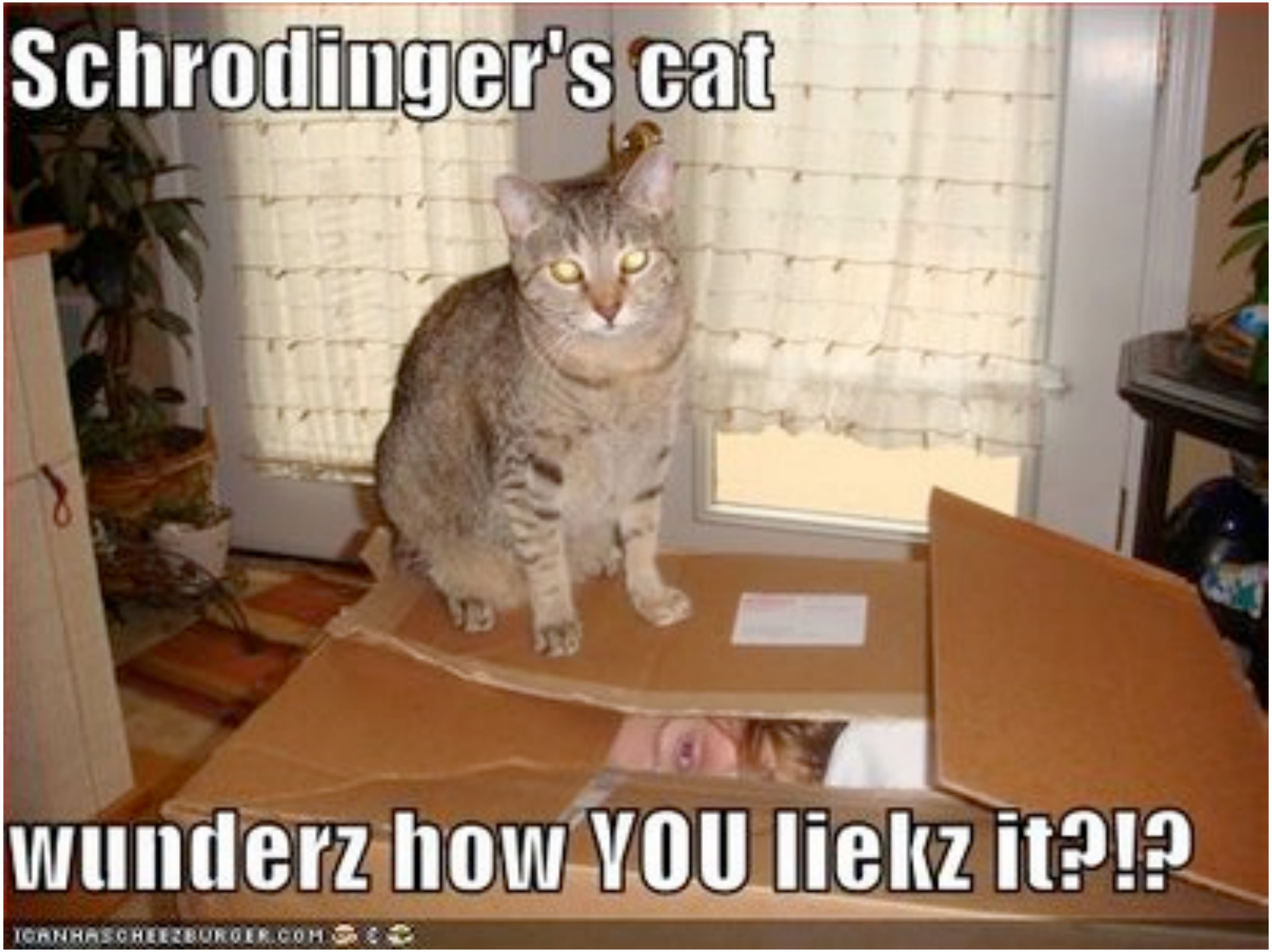
- Instrumentalist picture: particles exist at the moment of measurement, distributed according to the probability distribution. Statistical distribution of configurations is consistent with wave collapse, regardless of whether or not it occurs.

- Bohmian picture: particles exist for all time; in particular the particle we measure follows the trajectory q(x,t). The wave collapse approximation does not significantly alter q(x,t).

- GRW/Objective (Stochastic) Collapse: No comment.

Conclusion

- Derived multiconfiguration mean field model for quantum system consisting of a particle interacting with a Bose gas.
- Reduced model to classical particle coupled to a Bose gas.
- Derived quantum friction, showing that the particle eventually stops.
- Showed that statistical significance of experimental outcomes provides upper bound on quantum interference.
- Suggested possibilities for fractional measurements.



Thank you